Reverse weak formulation for quasi-static medical elastography inversion under the plane stress approximation and stability issues

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Abstract

This work focuses on the reconstruction of the shear modulus of biological tissues from quasi-static elastic internal displacement fields. The presented method is based on the Reverse Weak Formulation which consist in switching the unknown parameter and the displacement field in the variational formulation of the linear elastic system of PDE. This formulation allows us to consider the inverse parameter problem as a null space identification problem of a formal operator built from the data. Its main advantage is that it does not require boundary data nor high smoothness hypothesis.

In biological tissues, elastic deformations are three dimensional and the classical ultrasound data indirectly provides 2D displacement field in the imaging plane. A plane stress hypothesis is made to reduce the problem to a new 2D elastic problem depending only on the shear modulus and solved by a projection of the total field.

Theoretical stability results will be presented and the question of preserving this stability in the discrete version of the problem will be addressed. As we will see, this question is similar to classical problems involving the so called inf-sup constant and discrete inf-sup constant of the divergence operator (also called LBB constants). These constants are well known in the context of the discretization of incompressible Stokes problems for instance. These techniques are adapted here for a larger class of operators to provide discrete error estimates that depend on the choice of the discretization spaces.

The resulting method is efficient as it does not require iterative resolution of the forward problem, nor additional data at the boundary. It only requires to compute the first singular vector of the discretized operator. Several discretization spaces are numerically compared and the method is tested on synthetical, experimental and medical data in the context of breast tumor detection.

References

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