Asymptotic inverse problems for a depolymerising system

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Abstract

Shrinkage, either through depolymerisation (*i.e.*, progressive shortening) or through fragmentation (breakage into smaller pieces) are dynamical phenomena which appear in many applications. More specifically, the departure point of our research has been protein fibrils depolymerisation, thought to be a key mechanism for many diseases (Parkinson's, Alzheimer's...) as well as many functional biomolecular systems (actin filaments). The dynamic nature of the experiments, as well as their nanoscale, makes it very challenging to estimate their features.

In this talk, we consider a model of discrete depolymerisation, originally based on a Becker-Döring model. Our aim is to estimate the initial distribution when measuring the dynamics of the first or second moment of the distribution during the depolymerization process.

We first evaluate the impact of using continuous approximations of the initial discrete model to solve this inverse problem. At first order, the model is approximated by a backward transport equation, for which the inverse problem turns to be mildly ill-posed (of order $k+1$ when used to invert the time evolution of the k-moment of the solution). This remains true when polymerisation is also considered, as in the full Becker-Döring system, though the inversion reveals more intricate due to the fact that the problem becomes nonlinear.

At second order, the asymptotic model becomes an advection-diffusion equation, where the diffusion is a corrective term, complemented with an original transparent boundary condition at $x = 0$. This approximation is much more accurate, but we face a classical accuracy versus stability trade-off: the inverse reconstruction reveals to be severely ill-posed. Thanks to Carleman inequalities and to log-convexity estimates, we prove observability results and error estimates for a Tikhonov regularisation. We then develop a Kalman-based observer approach, which reveals very efficient for the numerical solution.

This is a joint work with Philippe Moireau (Inria), inspired by depolymerisation experiments carried out by Human Rezaei and collaborators (Inrae).